# NCERT Solutions for Class 11 Maths Chapter 3

## Trigonometric Functions Class 11

Chapter 3 Trigonometric Functions Exercise 3.1, 3.2, 3.3, 3.4, miscellaneous Solutions

**Exercise 3.1 :** Solutions of Questions on Page Number : **54**

**Q1 :**

**Find the radian measures corresponding to the following degree measures:**

**(i) 25° (ii) - 47° 30' (iii) 240° (iv) 520°**

**Answer :**

(i) 25°

We know that 180° = π radian

$$\therefore 25° = \frac{\pi}{180} \times 25 \text{ radian} = \frac{5\pi}{36} \text{ radian}$$

(ii) â€"47° 30'

â€"47° 30' = $-47\dfrac{1}{2}$ degree [1° = 60']

$$= \frac{-95}{2} \text{ degree}$$

Since 180° = π radian

$$\frac{-95}{2} \text{ deg ree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{ radian} = \left(\frac{-19}{36 \times 2}\right) \pi \text{ radian} = \frac{-19}{72} \pi \text{ radian}$$

$$\therefore -47° 30' = \frac{-19}{72} \pi \text{ radian}$$

(iii) 240°

We know that 180° = π radian

$$\therefore 240° = \frac{\pi}{180} \times 240 \text{ radian} = \frac{4}{3}\pi \text{ radian}$$

(iv) 520°

We know that 180° = π radian

$$\therefore 520° = \frac{\pi}{180} \times 520 \text{ radian} = \frac{26\pi}{9} \text{ radian}$$

**Q2 :**

**Find the degree measures corresponding to the following radian measures**

$$\left(\text{Use } \pi = \frac{22}{7}\right).$$

(i) $\dfrac{11}{16}$  (ii) – 4 (iii) $\dfrac{5\pi}{3}$  (iv) $\dfrac{7\pi}{6}$

**Answer :**

(i) $\dfrac{11}{16}$

We know that π radian = 180°

$$\therefore \frac{11}{16} \text{ radain} = \frac{180}{\pi} \times \frac{11}{16} \text{ deg ree} = \frac{45 \times 11}{\pi \times 4} \text{ deg ree}$$

$$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ deg ree} = \frac{315}{8} \text{ deg ree}$$

$$= 39\frac{3}{8} \text{ deg ree}$$

$$= 39° + \frac{3 \times 60}{8} \text{ min utes} \qquad [1° = 60']$$

$$= 39° + 22' + \frac{1}{2} \text{ min utes}$$

$$= 39°22'30'' \qquad [1' = 60'']$$

(ii) – 4

We know that π radian = 180°

$$-4 \text{ radian} = \frac{180}{\pi} \times (-4) \text{ deg ree} = \frac{180 \times 7(-4)}{22} \text{ deg ree}$$

$$= \frac{-2520}{11} \text{ deg ree} = -229\frac{1}{11} \text{ deg ree}$$

$$= -229° + \frac{1 \times 60}{11} \text{ min utes} \qquad [1° = 60']$$

$$= -229° + 5' + \frac{5}{11} \text{ min utes}$$

$$= -229°5'27'' \qquad [1' = 60'']$$

(iii) $\dfrac{5\pi}{3}$

We know that π radian = 180°

$$\therefore \frac{5\pi}{3} \text{radian} = \frac{180}{\pi} \times \frac{5\pi}{3} \text{ deg ree} = 300°$$

(iv) $\dfrac{7\pi}{6}$

We know that π radian = 180°

$$\therefore \frac{7\pi}{6} \text{radian} = \frac{180}{\pi} \times \frac{7\pi}{6} = 210°$$

**Q3 :**

**A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?**

**Answer :**

Number of revolutions made by the wheel in 1 minute = 360

∴Number of revolutions made by the wheel in 1 second = $\dfrac{360}{60} = 6$

In one complete revolution, the wheel turns an angle of 2π radian.

Hence, in 6 complete revolutions, it will turn an angle of 6 × 2π radian, i.e.,

12 π radian

Thus, in one second, the wheel turns an angle of 12π radian.

**Q4 :**

**Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm** $\left( \text{Use } \pi = \dfrac{22}{7} \right)$ **.**

**Answer :**

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre, then

$$\theta = \frac{l}{r}$$

Therefore, for r = 100 cm, l = 22 cm, we have

$$\theta = \frac{22}{100} \text{ radian} = \frac{180}{\pi} \times \frac{22}{100} \text{ deg ree} = \frac{180 \times 7 \times 22}{22 \times 100} \text{ deg ree}$$

$$= \frac{126}{10} \text{ deg ree} = 12\frac{3}{5} \text{ deg ree} = 12°36' \quad [1° = 60']$$

Thus, the required angle is 12°36"².
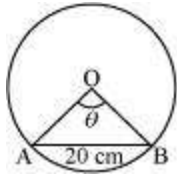
**Q5 :**

**In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.**

**Answer :**

Diameter of the circle = 40 cm

∴Radius ($r$) of the circle = $\dfrac{40}{2}$ cm = 20 cm

Let AB be a chord (length = 20 cm) of the circle.



In ΔOAB, OA = OB = Radius of circle = 20 cm

Also, AB = 20 cm

Thus, ΔOAB is an equilateral triangle.

∴θ = 60° = $\dfrac{\pi}{3}$ radian

We know that in a circle of radius $r$ unit, if an arc of length $l$ unit subtends an angle $\theta$ radian at the centre, then $\theta = \dfrac{l}{r}$.

$$\dfrac{\pi}{3} = \dfrac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \dfrac{20\pi}{3} \text{ cm}$$

Thus, the length of the minor arc of the chord is $\dfrac{20\pi}{3}$ cm.

**Q6 :**

**If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.**

**Answer :**

Let the radii of the two circles be $r_1$ and $r_2$. Let an arc of length $l$ subtend an angle of 60° at the centre of the circle of radius $r_1$, while let an arc of length $l$ subtend an angle of 75° at the centre of the circle of radius $r_2$.

Now, 60° = $\dfrac{\pi}{3}$ radian and 75° = $\dfrac{5\pi}{12}$ radian

We know that in a circle of radius $r$ unit, if an arc of length $l$ unit subtends an angle $\theta$ radian at the centre, then

$$\theta = \dfrac{l}{r} \text{ or } l = r\theta$$.

$$\therefore l = \dfrac{r_1\pi}{3} \text{ and } l = \dfrac{r_2 5\pi}{12}$$

$$\Rightarrow \dfrac{r_1\pi}{3} = \dfrac{r_2 5\pi}{12}$$

$$\Rightarrow r_1 = \dfrac{r_2 5}{4}$$

$$\Rightarrow \dfrac{r_1}{r_2} = \dfrac{5}{4}$$

Thus, the ratio of the radii is 5:4.

**Q7 :**

**Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length**

**(i) 10 cm (ii) 15 cm (iii) 21 cm**

**Answer :**

We know that in a circle of radius $r$ unit, if an arc of length $l$ unit subtends an angle $\theta$ radian at the centre, then $\theta = \dfrac{l}{r}$.

It is given that $r = 75$ cm

(i) Here, $l = 10$ cm

$$\theta = \dfrac{10}{75} \text{ radian} = \dfrac{2}{15} \text{ radian}$$

(ii) Here, $l = 15$ cm

$$\theta = \dfrac{15}{75} \text{ radian} = \dfrac{1}{5} \text{ radian}$$

(iii) Here, $l = 21$ cm

$$\theta = \dfrac{21}{75} \text{ radian} = \dfrac{7}{25} \text{ radian}$$

**Exercise 3.2 :** Solutions of Questions on Page Number : **63**

**Q1 :**

Find the values of other five trigonometric functions if $\cos x = -\dfrac{1}{2}$, $x$ lies in third quadrant.

**Answer :**

$$\cos x = -\frac{1}{2}$$

$$\therefore \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\dfrac{1}{2}\right)} = -2$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$$

Since $x$ lies in the 3ʳᵈ quadrant, the value of sin $x$ will be negative.

$$\therefore \sin x = -\frac{\sqrt{3}}{2}$$

$$\cos ec x = \frac{1}{\sin x} = \frac{1}{\left(-\dfrac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\dfrac{\sqrt{3}}{2}\right)}{\left(-\dfrac{1}{2}\right)} = \sqrt{3}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

**Q2 :**

**Find the values of other five trigonometric functions if** $\sin x = \dfrac{3}{5}$ **, x lies in second quadrant.**

**Answer :**

$\sin x = \dfrac{3}{5}$

$\operatorname{cosec} x = \dfrac{1}{\sin x} = \dfrac{1}{\left(\dfrac{3}{5}\right)} = \dfrac{5}{3}$

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \sin^2 x$

$\Rightarrow \cos^2 x = 1 - \left(\dfrac{3}{5}\right)^2$

$\Rightarrow \cos^2 x = 1 - \dfrac{9}{25}$

$\Rightarrow \cos^2 x = \dfrac{16}{25}$

$\Rightarrow \cos x = \pm \dfrac{4}{5}$

Since *x* lies in the 2nd quadrant, the value of cos *x* will be negative

$\therefore \cos x = -\dfrac{4}{5}$

$\sec x = \dfrac{1}{\cos x} = \dfrac{1}{\left(-\dfrac{4}{5}\right)} = -\dfrac{5}{4}$

$\tan x = \dfrac{\sin x}{\cos x} = \dfrac{\left(\dfrac{3}{5}\right)}{\left(-\dfrac{4}{5}\right)} = -\dfrac{3}{4}$

$\cot x = \dfrac{1}{\tan x} = -\dfrac{4}{3}$

**Q3 :**

**Find the values of other five trigonometric functions if** $\cot x = \dfrac{3}{4}$ **, x lies in third quadrant.**

**Answer :**

$$\cot x = \frac{3}{4}$$

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(\frac{4}{3}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$$

$$\Rightarrow \frac{25}{9} = \sec^2 x$$

$$\Rightarrow \sec x = \pm \frac{5}{3}$$

Since *x* lies in the 3ʳᵈ quadrant, the value of sec *x* will be negative.

$$\therefore \sec x = -\frac{5}{3}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$$

$$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

**Q4 :**

**Find the values of other five trigonometric functions if** $\sec x = \frac{13}{5}$ **, *x* lies in fourth quadrant.**

**Answer :**

$$\sec x = \frac{13}{5}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\Rightarrow \sin x = \pm \frac{12}{13}$$

Since $x$ lies in the 4ᵗʰ quadrant, the value of sin $x$ will be negative.

$$\therefore \sin x = -\frac{12}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

**Q5 :**

Find the values of other five trigonometric functions if $\tan x = -\frac{5}{12}$ , $x$ lies in second quadrant.

**Answer :**

$$\tan x = -\frac{5}{12}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\dfrac{5}{12}\right)} = -\frac{12}{5}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{25}{144} = \sec^2 x$$

$$\Rightarrow \frac{169}{144} = \sec^2 x$$

$$\Rightarrow \sec x = \pm \frac{13}{12}$$

Since $x$ lies in the 2$^{nd}$ quadrant, the value of sec $x$ will be negative.

$$\therefore \sec x = \frac{-\dfrac{13}{12}}{}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\dfrac{13}{12}\right)} = -\frac{12}{13}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow -\frac{5}{12} = \frac{\sin x}{\left(-\dfrac{12}{13}\right)}$$

$$\Rightarrow \sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

$$\csc x = \frac{1}{\sin x} = \frac{1}{\left(\dfrac{5}{13}\right)} = \frac{13}{5}$$

**Q6 :**

**Find the value of the trigonometric function sin 765°**

**Answer :**

It is known that the values of sin $x$ repeat after an interval of 2π or 360°.

$$\therefore \sin 765° = \sin(2 \times 360° + 45°) = \sin 45° = \frac{1}{\sqrt{2}}$$

**Q7 :**

**Find the value of the trigonometric function cosec (-1410°)**

**Answer :**

It is known that the values of cosec $x$ repeat after an interval of $2\pi$ or 360°.

$$\therefore \text{cosec} \ (-1410°) = \text{cosec} \ (-1410° + 4 \times 360°)$$
$$= \text{cosec} \ (-1410° + 1440°)$$
$$= \text{cosec} 30° = 2$$

**Q8 :**

**Find the value of the trigonometric function** $\tan \dfrac{19\pi}{3}$

**Answer :**

It isknown that the values of tan $x$ repeat after an interval of $\pi$ or 180°.

$$\therefore \tan \frac{19\pi}{3} = \tan 6\frac{1}{3}\pi = \tan \left( 6\pi + \frac{\pi}{3} \right) = \tan \frac{\pi}{3} = \tan 60° = \sqrt{3}$$

**Q9 :**

**Find the value of the trigonometric function** $\sin \left( -\dfrac{11\pi}{3} \right)$

**Answer :**

It is known that the values of sin $x$ repeat after an interval of $2\pi$ or 360°.

$$\therefore \sin \left( -\frac{11\pi}{3} \right) = \sin \left( -\frac{11\pi}{3} + 2 \times 2\pi \right) = \sin \left( \frac{\pi}{3} \right) = \frac{\sqrt{3}}{2}$$

**Q10 :**

**Find the value of the trigonometric function** $\cot \left( -\dfrac{15\pi}{4} \right)$

**Answer :**

It is known that the values of cot $x$ repeat after an interval of π or 180°.

$$\therefore \cot\left(-\frac{15\pi}{4}\right) = \cot\left(-\frac{15\pi}{4} + 4\pi\right) = \cot\frac{\pi}{4} = 1$$

**Exercise 3.3 :** Solutions of Questions on Page Number : **73**

**Q1 :**

$$\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = -\frac{1}{2}$$

**Answer :**

L.H.S. = $\sin^2\dfrac{\pi}{6} + \cos^2\dfrac{\pi}{3} - \tan^2\dfrac{\pi}{4}$

$= \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2 - (1)^2$

$= \dfrac{1}{4} + \dfrac{1}{4} - 1 = -\dfrac{1}{2}$

$= $ R.H.S.

**Q2 :**

**Prove that** $2\sin^2\dfrac{\pi}{6} + \operatorname{cosec}^2\dfrac{7\pi}{6}\cos^2\dfrac{\pi}{3} = \dfrac{3}{2}$

**Answer :**

L.H.S. = $2\sin^2\dfrac{\pi}{6} + \operatorname{cosec}^2\dfrac{7\pi}{6}\cos^2\dfrac{\pi}{3}$

$$= 2\left(\frac{1}{2}\right)^2 + \cos ec^2\left(\pi + \frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$$

$$= 2 \times \frac{1}{4} + \left(-\cos ec \frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$

$$= \frac{1}{2} + (-2)^2\left(\frac{1}{4}\right)$$

$$= \frac{1}{2} + \frac{4}{4} = \frac{1}{2} + 1 = \frac{3}{2}$$

$$= \text{R.H.S.}$$

**Q3 :**

**Prove that** $\cot^2 \frac{\pi}{6} + \cos ec \frac{5\pi}{6} + 3\tan^2 \frac{\pi}{6} = 6$

**Answer :**

L.H.S. = $\cot^2 \frac{\pi}{6} + \cos ec \frac{5\pi}{6} + 3\tan^2 \frac{\pi}{6}$

$$= \left(\sqrt{3}\right)^2 + \cos ec\left(\pi - \frac{\pi}{6}\right) + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \cos ec \frac{\pi}{6} + 3 \times \frac{1}{3}$$

$$= 3 + 2 + 1 = 6$$

$$= \text{R.H.S}$$

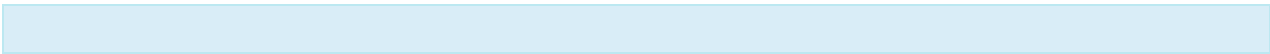**Q4 :**

**Prove that** $2\sin^2 \frac{3\pi}{4} + 2\cos^2 \frac{\pi}{4} + 2\sec^2 \frac{\pi}{3} = 10$

**Answer :**

L.H.S = $2\sin^2 \frac{3\pi}{4} + 2\cos^2 \frac{\pi}{4} + 2\sec^2 \frac{\pi}{3}$

$$= 2\left\{ \sin\left(\pi - \frac{\pi}{4}\right)\right\}^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$$

$$= 2\left\{ \sin\frac{\pi}{4}\right\}^2 + 2\times\frac{1}{2} + 8$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8$$

$$= 1 + 1 + 8$$

$$= 10$$

$$= \text{R.H.S}$$

**Q5 :**

**Find the value of:**

**(i) sin 75°**

**(ii) tan 15°**

**Answer :**

(i) sin 75° = sin (45° + 30°)

= sin 45° cos 30° + cos 45° sin 30°

[sin (x + y) = sin x cos y + cos x sin y]

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) tan 15° = tan (45° – 30°)

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} \qquad \left[ \tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \right]$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)} = \frac{3+1-2\sqrt{3}}{\left(\sqrt{3}\right)^2 - (1)^2}$$

$$= \frac{4-2\sqrt{3}}{3-1} = 2 - \sqrt{3}$$

**Q6 :**

**Prove that:** $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right) = \sin(x+y)$

**Answer :**

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$$

$$= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)\right] + \frac{1}{2}\left[-2\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)\right]$$

$$= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\} + \cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$+ \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\} - \cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$\begin{bmatrix} \because 2\cos A \cos B = \cos(A+B) + \cos(A-B) \\ -2\sin A \sin B = \cos(A+B) - \cos(A-B) \end{bmatrix}$$

$$= 2 \times \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$= \cos\left[\frac{\pi}{2}-(x+y)\right]$$

$$= \sin(x+y)$$

$$= \text{R.H.S}$$

**Q7 :**

**Prove that:** $\dfrac{\tan\left(\dfrac{\pi}{4}+x\right)}{\tan\left(\dfrac{\pi}{4}-x\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^{2}$

**Answer :**

It is known that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ and $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

∴L.H.S. = $\dfrac{\tan\left(\dfrac{\pi}{4}+x\right)}{\tan\left(\dfrac{\pi}{4}-x\right)}=\dfrac{\left(\dfrac{\tan\dfrac{\pi}{4}+\tan x}{1-\tan\dfrac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\dfrac{\pi}{4}-\tan x}{1+\tan\dfrac{\pi}{4}\tan x}\right)}=\dfrac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^{2}=$ R.H.S.

**Q8 :**

**Prove that** $\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\dfrac{\pi}{2}+x\right)}=\cot^{2}x$

**Answer :**

$$\text{L.H.S.} = \dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\dfrac{\pi}{2}+x\right)}$$

$$= \dfrac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$

$$= \dfrac{-\cos^{2}x}{-\sin^{2}x}$$

$$= \cot^{2}x$$

$$= \text{R.H.S.}$$

**Q9 :**

$$\cos\left(\frac{3\pi}{2}+x\right)\cos\left(2\pi+x\right)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot\left(2\pi+x\right)\right]=1$$

**Answer :**

L.H.S. = $\cos\left(\dfrac{3\pi}{2}+x\right)\cos\left(2\pi+x\right)\left[\cot\left(\dfrac{3\pi}{2}-x\right)+\cot\left(2\pi+x\right)\right]$

$=\sin x\cos x\left[\tan x+\cot x\right]$

$=\sin x\cos x\left(\dfrac{\sin x}{\cos x}+\dfrac{\cos x}{\sin x}\right)$

$=\left(\sin x\cos x\right)\left[\dfrac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right]$

$=1=$ R.H.S.

**Q10 :**

**Prove that sin (*n* + 1)*x* sin (*n* + 2)*x* + cos (*n* + 1)*x* cos (*n* + 2)*x* = cos *x***

**Answer :**

L.H.S. = sin (*n* + 1)*x* sin(*n* + 2)*x* + cos (*n* + 1)*x* cos(*n* + 2)*x*

$=\dfrac{1}{2}\left[2\sin\left(n+1\right)x\sin\left(n+2\right)x+2\cos\left(n+1\right)x\cos\left(n+2\right)x\right]$

$=\dfrac{1}{2}\left[\begin{array}{l}\cos\{(n+1)x-(n+2)x\}-\cos\{(n+1)x+(n+2)x\}\\+\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\}\end{array}\right]$

$\left[\begin{array}{l}\because -2\sin A\sin B=\cos\left(A+B\right)-\cos\left(A-B\right)\\2\cos A\cos B=\cos\left(A+B\right)+\cos\left(A-B\right)\end{array}\right]$

$=\dfrac{1}{2}\times 2\cos\{(n+1)x-(n+2)x\}$

$=\cos\left(-x\right)=\cos x=$ R.H.S.

**Q11 :**

**Prove that**
$$\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$$

**Answer :**

It is known that
$$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right).$$

∴L.H.S. =
$$\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$$

$$=-2\sin\left\{\frac{\left[\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)\right]}{2}\right\}.\sin\left\{\frac{\left[\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)\right]}{2}\right\}$$

$$=-2\sin\left(\frac{3\pi}{4}\right)\sin x$$

$$=-2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

$$=-2\sin\frac{\pi}{4}\sin x$$

$$=-2\times\frac{1}{\sqrt{2}}\times\sin x$$

$$=-\sqrt{2}\sin x$$

= R.H.S.

**Q12 :**

**Prove that sin² 6x - sin² 4x = sin 2x sin 10x**

**Answer :**

It is known that
$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\quad \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

∴L.H.S. = sin²6x – sin²4x

= (sin 6x + sin 4x) (sin 6x – sin 4x)

$$=\left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$$

= (2 sin 5*x* cos *x*) (2 cos 5*x* sin *x*)

= (2 sin 5*x* cos 5*x*) (2 sin *x* cos *x*)

= sin 10*x* sin 2*x*

= R.H.S.

**Q13 :**

**Prove that cos² 2*x* - cos² 6*x* = sin 4*x* sin 8*x***

**Answer :**

It is known
that
$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \quad \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

∴L.H.S. = cos² 2*x* – cos² 6*x*

= (cos 2*x* + cos 6*x*) (cos 2*x* – 6*x*)

$$= \left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$$

$$= \left[2\cos 4x \cos(-2x)\right]\left[-2\sin 4x \sin(-2x)\right]$$

= [2 cos 4*x* cos 2*x*] [–2 sin 4*x* (–sin 2*x*)]

= (2 sin 4*x* cos 4*x*) (2 sin 2*x* cos 2*x*)

= sin 8*x* sin 4*x*

= R.H.S.

**Q14 :**

**Prove that sin 2*x* + 2sin 4*x* + sin 6*x* = 4cos² *x* sin 4*x***

**Answer :**

L.H.S. = sin 2*x* + 2 sin 4*x* + sin 6*x*

= [sin 2*x* + sin 6*x*] + 2 sin 4*x*

$$= \left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right] + 2\sin 4x$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

= 2 sin 4*x* cos (– 2*x*) + 2 sin 4*x*

= 2 sin $4x$ cos $2x$ + 2 sin $4x$

= 2 sin $4x$ (cos $2x$ + 1)

= 2 sin $4x$ (2 cos² $x$ – 1 + 1)

= 2 sin $4x$ (2 cos² $x$)

= 4cos² $x$ sin $4x$

= R.H.S.

**Q15 :**

**Prove that cot $4x$ (sin $5x$ + sin $3x$) = cot $x$ (sin $5x$ - sin $3x$)**

**Answer :**

L.H.S = cot $4x$ (sin $5x$ + sin $3x$)

$$= \frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \left(\frac{\cos 4x}{\sin 4x}\right)\left[2\sin 4x\cos x\right]$$

= 2 cos $4x$ cos $x$

R.H.S. = cot $x$ (sin $5x$ – sin $3x$)

$$= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{\cos x}{\sin x}\left[2\cos 4x\sin x\right]$$

= 2 cos $4x$. cos $x$

L.H.S. = R.H.S.

**Q16 :**

**Prove that** $\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\dfrac{\sin 2x}{\cos 10x}$

**Answer :**

It is known that

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \quad \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\therefore L.H.S = \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$$

$$= \frac{-2\sin\left(\frac{9x+5x}{2}\right).\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right).\sin\left(\frac{17x-3x}{2}\right)}$$

$$= \frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$$

$$= -\frac{\sin 2x}{\cos 10x}$$

$$= R.H.S.$$

**Q17 :**

Prove that $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

**Answer :**

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore L.H.S. = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

$$= \frac{2\sin\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$$

$$= \frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$$

$$= \frac{\sin 4x}{\cos 4x}$$

$$= \tan 4x = R.H.S.$$

**Q18 :**

**Prove that** $\dfrac{\sin x - \sin y}{\cos x + \cos y} = \tan \dfrac{x-y}{2}$

**Answer :**

It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

∴L.H.S. = $\dfrac{\sin x - \sin y}{\cos x + \cos y}$

$$= \frac{2\cos\left(\dfrac{x+y}{2}\right).\sin\left(\dfrac{x-y}{2}\right)}{2\cos\left(\dfrac{x+y}{2}\right).\cos\left(\dfrac{x-y}{2}\right)}$$

$$= \frac{\sin\left(\dfrac{x-y}{2}\right)}{\cos\left(\dfrac{x-y}{2}\right)}$$

$$= \tan\left(\frac{x-y}{2}\right) = \text{R.H.S.}$$

**Q19 :**

**Prove that** $\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

**Answer :**

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

∴L.H.S. = $\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x}$

$$= \frac{2\sin\left(\dfrac{x+3x}{2}\right)\cos\left(\dfrac{x-3x}{2}\right)}{2\cos\left(\dfrac{x+3x}{2}\right)\cos\left(\dfrac{x-3x}{2}\right)}$$

$$= \frac{\sin 2x}{\cos 2x}$$

$$= \tan 2x$$

$$= \text{R.H.S}$$

**Q20 :**

**Prove that** $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

**Answer :**

It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \quad \cos^2 A - \sin^2 A = \cos 2A$$

$\therefore$ L.H.S. $= \dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$

$$= \frac{2\cos\left(\dfrac{x+3x}{2}\right)\sin\left(\dfrac{x-3x}{2}\right)}{-\cos 2x}$$

$$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$$

$$= -2 \times (-\sin x)$$

$$= 2\sin x = \text{R.H.S.}$$

**Q21 :**

**Prove that** $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

**Answer :**

L.H.S. $= \dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

$$\left[ \because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \ \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \right]$$

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

$$= \frac{\cos 3x (2\cos x + 1)}{\sin 3x (2\cos x + 1)}$$

$$= \cot 3x = \text{R.H.S.}$$

**Q22 :**

**Prove that cot $x$ cot $2x$ - cot $2x$ cot $3x$ - cot $3x$ cot $x$ = 1**

**Answer :**

L.H.S. = cot $x$ cot $2x$ – cot $2x$ cot $3x$ – cot $3x$ cot $x$

= cot $x$ cot $2x$ – cot $3x$ (cot $2x$ + cot $x$)

= cot $x$ cot $2x$ – cot $(2x + x)$ (cot $2x$ + cot $x$)

$$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x}\right](\cot 2x + \cot x)$$

$$\left[ \because \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B} \right]$$

= cot $x$ cot $2x$ – (cot $2x$ cot $x$ – 1)

= 1 = R.H.S.

**Q23 :**

$$\tan 4x = \frac{4\tan x \left(1 - \tan^2 x\right)}{1 - 6\tan^2 x + \tan^4 x}$$

**Prove that**

**Answer :**

It is known that $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$.

∴L.H.S. = tan 4x = tan 2(2x)

$= \dfrac{2\tan 2x}{1-\tan^2(2x)}$

$= \dfrac{2\left(\dfrac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\dfrac{2\tan x}{1-\tan^2 x}\right)^2}$

$= \dfrac{\left(\dfrac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\dfrac{4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$

$= \dfrac{\left(\dfrac{4\tan x}{1-\tan^2 x}\right)}{\left[\dfrac{\left(1-\tan^2 x\right)^2-4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$

$= \dfrac{4\tan x\left(1-\tan^2 x\right)}{\left(1-\tan^2 x\right)^2-4\tan^2 x}$

$= \dfrac{4\tan x\left(1-\tan^2 x\right)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$

$= \dfrac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x} = $ R.H.S.

**Q24 :**

**Prove that cos 4x = 1 - 8sin² x cos² x**

**Answer :**

L.H.S. = cos 4x

= cos 2(2x)

= 1 - 2 sin² 2x [cos 2A = 1 - 2 sin² A]

= 1 - 2(2 sin x cos x)² [sin2A = 2sin A cosA]

= 1 - 8 sin²x cos²x

= R.H.S.

**Q25 :**

**Prove that: cos 6$x$ = 32 cos$^6$ $x$ - 48 cos$^4$ $x$ + 18 cos$^2$ $x$ - 1**

**Answer :**

L.H.S. = cos 6$x$

= cos 3(2$x$)

= 4 cos$^3$ 2$x$ - 3 cos 2$x$ [cos 3$A$ = 4 cos$^3$ $A$ - 3 cos $A$]

= 4 [(2 cos$^2$ $x$ - 1)$^3$ - 3 (2 cos$^2$ $x$ - 1) [cos 2$x$ = 2 cos$^2$ $x$ - 1]

= 4 [(2 cos$^2$ $x$)$^3$ - (1)$^3$ - 3 (2 cos$^2$ $x$)$^2$ + 3 (2 cos$^2$ $x$)] - 6cos$^2$ $x$ + 3

= 4 [8cos$^6$$x$ - 1 - 12 cos$^4$$x$ + 6 cos$^2$$x$] - 6 cos$^2$$x$ + 3

= 32 cos$^6$$x$ - 4 - 48 cos$^4$$x$ + 24 cos$^2$ $x$ - 6 cos$^2$$x$ + 3

= 32 cos$^6$$x$ - 48 cos$^4$$x$ + 18 cos$^2$$x$ - 1

= R.H.S.

**Exercise 3.4 :** Solutions of Questions on Page Number : **78**

**Q1 :**

**Find the principal and general solutions of the equation** $\tan x = \sqrt{3}$

**Answer :**

$\tan x = \sqrt{3}$

It is known that $\tan \dfrac{\pi}{3} = \sqrt{3}$ and $\tan\left(\dfrac{4\pi}{3}\right) = \tan\left(\pi + \dfrac{\pi}{3}\right) = \tan \dfrac{\pi}{3} = \sqrt{3}$

Therefore, the principal solutions are $x = \dfrac{\pi}{3}$ and $\dfrac{4\pi}{3}$.

Now, $\tan x = \tan \dfrac{\pi}{3}$

$\Rightarrow x = n\pi + \dfrac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \dfrac{\pi}{3}$, where $n \in Z$

**Q2 :**

**Find the principal and general solutions of the equation** $\sec x = 2$

**Answer :**

$\sec x = 2$

It is known that $\sec \dfrac{\pi}{3} = 2$ and $\sec \dfrac{5\pi}{3} = \sec\left(2\pi - \dfrac{\pi}{3}\right) = \sec\dfrac{\pi}{3} = 2$

Therefore, the principal solutions are $x = \dfrac{\pi}{3}$ and $\dfrac{5\pi}{3}$ .

Now, $\sec x = \sec \dfrac{\pi}{3}$

$\Rightarrow \cos x = \cos \dfrac{\pi}{3}$         $\left[\sec x = \dfrac{1}{\cos x}\right]$

$\Rightarrow x = 2n\pi \pm \dfrac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = 2n\pi \pm \dfrac{\pi}{3}$ , where $n \in \mathbf{Z}$

**Q3 :**

**Find the principal and general solutions of the equation** $\cot x = -\sqrt{3}$

**Answer :**

$\cot x = -\sqrt{3}$

It is known that $\cot \dfrac{\pi}{6} = \sqrt{3}$

$\therefore \cot\left(\pi - \dfrac{\pi}{6}\right) = -\cot\dfrac{\pi}{6} = -\sqrt{3}$ and $\cot\left(2\pi - \dfrac{\pi}{6}\right) = -\cot\dfrac{\pi}{6} = -\sqrt{3}$

i.e., $\cot\dfrac{5\pi}{6} = -\sqrt{3}$ and $\cot\dfrac{11\pi}{6} = -\sqrt{3}$

Therefore, the principal solutions are $x = \dfrac{5\pi}{6}$ and $\dfrac{11\pi}{6}$. .

Now, $\cot x = \cot \dfrac{5\pi}{6}$

$\Rightarrow \tan x = \tan \dfrac{5\pi}{6}$ $\qquad \left[ \cot x = \dfrac{1}{\tan x} \right]$

$\Rightarrow x = n\pi + \dfrac{5\pi}{6},$ where $n \in Z$

Therefore, the general solution is $\qquad x = n\pi + \dfrac{5\pi}{6},$ where $n \in Z$

**Q4 :**

**Find the general solution of cosec $x = -2$**

**Answer :**

cosec x= â€"2

It is known that

$\operatorname{cosec} \dfrac{\pi}{6} = 2$

$\therefore \operatorname{cosec}\left(\pi + \dfrac{\pi}{6}\right) = -\operatorname{cosec}\dfrac{\pi}{6} = -2$ and $\operatorname{cosec}\left(2\pi - \dfrac{\pi}{6}\right) = -\operatorname{cosec}\dfrac{\pi}{6} = -2$

i.e., $\operatorname{cosec}\dfrac{7\pi}{6} = -2$ and $\operatorname{cosec}\dfrac{11\pi}{6} = -2$

Therefore, the principal solutions are $x = \dfrac{7\pi}{6}$ and $\dfrac{11\pi}{6}$.

Now, $\operatorname{cosec} x = \operatorname{cosec}\dfrac{7\pi}{6}$

$\Rightarrow \sin x = \sin \dfrac{7\pi}{6}$ $\qquad \left[ \operatorname{cosec} x = \dfrac{1}{\sin x} \right]$

$\Rightarrow x = n\pi + (-1)^n \dfrac{7\pi}{6},$ where $n \in Z$

Therefore, the general solution is $\qquad x = n\pi + (-1)^n \dfrac{7\pi}{6},$ where $n \in Z$

**Q5 :**

**Find the general solution of the equation** $\cos 4x = \cos 2x$

**Answer :**

$$\cos 4x = \cos 2x$$

$$\Rightarrow \cos 4x - \cos 2x = 0$$

$$\Rightarrow -2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$$

$$\left[\because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$\Rightarrow \sin 3x \sin x = 0$$

$$\Rightarrow \sin 3x = 0 \quad \text{or} \quad \sin x = 0$$

$$\therefore 3x = n\pi \quad \text{or} \quad x = n\pi, \text{ where } n \in Z$$

$$\Rightarrow x = \frac{n\pi}{3} \quad \text{or} \quad x = n\pi, \text{ where } n \in Z$$

**Q6 :**

**Find the general solution of the equation** $\cos 3x + \cos x - \cos 2x = 0$

**Answer :**

$$\cos 3x + \cos x - \cos 2x = 0$$

$$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0 \quad \left[\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$$

$$\Rightarrow \cos 2x(2\cos x - 1) = 0$$

$$\Rightarrow \cos 2x = 0 \quad \text{or} \quad 2\cos x - 1 = 0$$

$$\Rightarrow \cos 2x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$$

$$\therefore 2x = (2n+1)\frac{\pi}{2} \quad \text{or} \quad \cos x = \cos\frac{\pi}{3}, \text{ where } n \in Z$$

$$\Rightarrow x = (2n+1)\frac{\pi}{4} \quad \text{or} \quad x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in Z$$

**Q7 :**

**Find the general solution of the equation** $\sin 2x + \cos x = 0$

**Answer :**

$\sin 2x + \cos x = 0$

$\Rightarrow 2\sin x \cos x + \cos x = 0$

$\Rightarrow \cos x (2\sin x + 1) = 0$

$\Rightarrow \cos x = 0 \quad$ or $\quad 2\sin x + 1 = 0$

Now, $\cos x = 0 \Rightarrow \cos x = (2n+1)\dfrac{\pi}{2}$, where $n \in Z$

$2\sin x + 1 = 0$

$\Rightarrow \sin x = \dfrac{-1}{2} = -\sin\dfrac{\pi}{6} = \sin\left(\pi + \dfrac{\pi}{6}\right) = \sin\left(\pi + \dfrac{\pi}{6}\right) = \sin\dfrac{7\pi}{6}$

$\Rightarrow x = n\pi + (-1)^n \dfrac{7\pi}{6}$, where $n \in Z$

Therefore, the general solution is $(2n+1)\dfrac{\pi}{2}$ or $n\pi + (-1)^n \dfrac{7\pi}{6}, n \in Z$ .

**Q8 :**

**Find the general solution of the equation** $\sec^2 2x = 1 - \tan 2x$

**Answer :**

$\sec^2 2x = 1 - \tan 2x$

$\Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$

$\Rightarrow \tan^2 2x + \tan 2x = 0$

$\Rightarrow \tan 2x (\tan 2x + 1) = 0$

$\Rightarrow \tan 2x = 0 \qquad$ or $\qquad \tan 2x + 1 = 0$

Now, $\tan 2x = 0$

$\Rightarrow \tan 2x = \tan 0$

$\Rightarrow 2x = n\pi + 0$, where $n \in Z$

$\Rightarrow x = \dfrac{n\pi}{2}$, where $n \in Z$

$\tan 2x + 1 = 0$

$\Rightarrow \tan 2x = -1 = -\tan\dfrac{\pi}{4} = \tan\left(\pi - \dfrac{\pi}{4}\right) = \tan\dfrac{3\pi}{4}$

$\Rightarrow 2x = n\pi + \dfrac{3\pi}{4}$, where $n \in Z$

$\Rightarrow x = \dfrac{n\pi}{2} + \dfrac{3\pi}{8}$, where $n \in Z$

Therefore, the general solution is $\dfrac{n\pi}{2}$ or $\dfrac{n\pi}{2} + \dfrac{3\pi}{8}$, $n \in Z$ .

**Q9 :**

**Find the general solution of the equation** $\sin x + \sin 3x + \sin 5x = 0$

**Answer :**

$\sin x + \sin 3x + \sin 5x = 0$

$(\sin x + \sin 5x) + \sin 3x = 0$

$\Rightarrow \left[2\sin\left(\dfrac{x+5x}{2}\right)\cos\left(\dfrac{x-5x}{2}\right)\right] + \sin 3x = 0 \qquad \left[\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)\right]$

$\Rightarrow 2\sin 3x \cos(-2x) + \sin 3x = 0$

$\Rightarrow 2\sin 3x \cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x(2\cos 2x + 1) = 0$

$\Rightarrow \sin 3x = 0 \qquad \text{or} \qquad 2\cos 2x + 1 = 0$

Now, $\sin 3x = 0 \Rightarrow 3x = n\pi$, where $n \in Z$

i.e., $x = \dfrac{n\pi}{3}$, where $n \in Z$

$2\cos 2x + 1 = 0$

$\Rightarrow \cos 2x = \dfrac{-1}{2} = -\cos\dfrac{\pi}{3} = \cos\left(\pi - \dfrac{\pi}{3}\right)$

$\Rightarrow \cos 2x = \cos\dfrac{2\pi}{3}$

$\Rightarrow 2x = 2n\pi \pm \dfrac{2\pi}{3}$, where $n \in Z$

$\Rightarrow x = n\pi \pm \dfrac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $\dfrac{n\pi}{3}$ or $n\pi \pm \dfrac{\pi}{3}$, $n \in Z$

**Exercise Miscellaneous :** Solutions of Questions on Page Number : **81**

**Q1 :**

**Prove that:** $2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13} = 0$

**Answer :**

L.H.S.

$= 2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= 2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} + 2\cos\left(\dfrac{\dfrac{3\pi}{13} + \dfrac{5\pi}{13}}{2}\right)\cos\left(\dfrac{\dfrac{3\pi}{13} - \dfrac{5\pi}{13}}{2}\right)$ $\left[\cos x + \cos y = 2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)\right]$

$= 2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} + 2\cos\dfrac{4\pi}{13}\cos\left(\dfrac{-\pi}{13}\right)$

$= 2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} + 2\cos\dfrac{4\pi}{13}\cos\dfrac{\pi}{13}$

$= 2\cos\dfrac{\pi}{13}\left[\cos\dfrac{9\pi}{13} + \cos\dfrac{4\pi}{13}\right]$

$= 2\cos\dfrac{\pi}{13}\left[2\cos\left(\dfrac{\dfrac{9\pi}{13} + \dfrac{4\pi}{13}}{2}\right)\cos\left(\dfrac{\dfrac{9\pi}{13} - \dfrac{4\pi}{13}}{2}\right)\right]$

$= 2\cos\dfrac{\pi}{13}\left[2\cos\dfrac{\pi}{2}\cos\dfrac{5\pi}{26}\right]$

$= 2\cos\dfrac{\pi}{13} \times 2 \times 0 \times \cos\dfrac{5\pi}{26}$

$= 0 = $ R.H.S

**Q2 :**

**Prove that:** (sin 3*x* + sin *x*) sin *x* + (cos 3*x* - cos *x*) cos *x* = 0

**Answer :**

L.H.S.

= (sin 3x + sin x) sin x + (cos 3x – cos x) cos x

$$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x$$

$$= \cos 3x \cos x + \sin 3x \sin x - \left(\cos^2 x - \sin^2 x\right)$$

$$= \cos(3x - x) - \cos 2x \qquad \left[\cos(A - B) = \cos A \cos B + \sin A \sin B\right]$$

$$= \cos 2x - \cos 2x$$

$$= 0$$

= RH.S.

**Q3 :**

**Prove that:** $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2 \dfrac{x + y}{2}$

**Answer :**

L.H.S. = $(\cos x + \cos y)^2 + (\sin x - \sin y)^2$

$$= \cos^2 x + \cos^2 y + 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$$

$$= \left(\cos^2 x + \sin^2 x\right) + \left(\cos^2 y + \sin^2 y\right) + 2\left(\cos x \cos y - \sin x \sin y\right)$$

$$= 1 + 1 + 2\cos(x + y) \qquad \left[\cos(A + B) = (\cos A \cos B - \sin A \sin B)\right]$$

$$= 2 + 2\cos(x + y)$$

$$= 2\left[1 + \cos(x + y)\right]$$

$$= 2\left[1 + 2\cos^2\left(\frac{x + y}{2}\right) - 1\right] \qquad \left[\cos 2A = 2\cos^2 A - 1\right]$$

$$= 4\cos^2\left(\frac{x + y}{2}\right) = \text{R.H.S.}$$

**Q4 :**

**Prove that:** $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \dfrac{x - y}{2}$

**Answer :**

L.H.S. = $(\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$$

$$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2[\cos x \cos y + \sin x \sin y]$$

$$= 1 + 1 - 2[\cos(x - y)] \qquad [\cos(A - B) = \cos A \cos B + \sin A \sin B]$$

$$= 2[1 - \cos(x - y)]$$

$$= 2\left[1 - \left\{1 - 2\sin^2\left(\frac{x-y}{2}\right)\right\}\right] \qquad [\cos 2A = 1 - 2\sin^2 A]$$

$$= 4\sin^2\left(\frac{x-y}{2}\right) = \text{R.H.S.}$$

**Q5 :**

**Prove that:** $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$

**Answer :**

It is known that $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$.

$\therefore$ L.H.S. $= \sin x + \sin 3x + \sin 5x + \sin 7x$

$$= (\sin x + \sin 5x) + (\sin 3x + \sin 7x)$$

$$= 2\sin\left(\frac{x + 5x}{2}\right) \cdot \cos\left(\frac{x - 5x}{2}\right) + 2\sin\left(\frac{3x + 7x}{2}\right)\cos\left(\frac{3x - 7x}{2}\right)$$

$$= 2\sin 3x \cos(-2x) + 2\sin 5x \cos(-2x)$$

$$= 2\sin 3x \cos 2x + 2\sin 5x \cos 2x$$

$$= 2\cos 2x [\sin 3x + \sin 5x]$$

$$= 2\cos 2x \left[2\sin\left(\frac{3x + 5x}{2}\right) \cdot \cos\left(\frac{3x - 5x}{2}\right)\right]$$

$$= 2\cos 2x [2\sin 4x \cdot \cos(-x)]$$

$$= 4\cos 2x \sin 4x \cos x = \text{R.H.S.}$$

**Q6 :**

**Prove that:** $\dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

**Answer :**

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$$

L.H.S. $= \dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$

$$= \frac{\left[2\sin\left(\dfrac{7x+5x}{2}\right)\cdot\cos\left(\dfrac{7x-5x}{2}\right)\right] + \left[2\sin\left(\dfrac{9x+3x}{2}\right)\cdot\cos\left(\dfrac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\dfrac{7x+5x}{2}\right)\cdot\cos\left(\dfrac{7x-5x}{2}\right)\right] + \left[2\cos\left(\dfrac{9x+3x}{2}\right)\cdot\cos\left(\dfrac{9x-3x}{2}\right)\right]}$$

$$= \frac{[2\sin 6x \cdot \cos x] + [2\sin 6x \cdot \cos 3x]}{[2\cos 6x \cdot \cos x] + [2\cos 6x \cdot \cos 3x]}$$

$$= \frac{2\sin 6x[\cos x + \cos 3x]}{2\cos 6x[\cos x + \cos 3x]}$$

$= \tan 6x$

$= $ R.H.S.

**Q7 :**

**Prove that:** $\sin 3x + \sin 2x - \sin x = 4\sin x \cos\dfrac{x}{2}\cos\dfrac{3x}{2}$

**Answer :**

L.H.S. $= \sin 3x + \sin 2x - \sin x$

$$= \sin 3x + (\sin 2x - \sin x)$$

$$= \sin 3x + \left[ 2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right) \right] \qquad \left[ \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \right]$$

$$= \sin 3x + \left[ 2\cos\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right) \right]$$

$$= \sin 3x + 2\cos\frac{3x}{2}\sin\frac{x}{2}$$

$$= 2\sin\frac{3x}{2}\cdot\cos\frac{3x}{2} + 2\cos\frac{3x}{2}\sin\frac{x}{2} \qquad [\sin 2A = 2\sin A\cdot\cos B]$$

$$= 2\cos\left(\frac{3x}{2}\right)\left[ \sin\left(\frac{3x}{2}\right) + \sin\left(\frac{x}{2}\right) \right]$$

$$= 2\cos\left(\frac{3x}{2}\right)\left[ 2\sin\left\{\frac{\left(\frac{3x}{2}\right)+\left(\frac{x}{2}\right)}{2}\right\}\cos\left\{\frac{\left(\frac{3x}{2}\right)-\left(\frac{x}{2}\right)}{2}\right\} \right] \qquad \left[ \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \right]$$

$$= 2\cos\left(\frac{3x}{2}\right)\cdot 2\sin x\cos\left(\frac{x}{2}\right)$$

$$= 4\sin x\cos\left(\frac{x}{2}\right)\cos\left(\frac{3x}{2}\right) = \text{R.H.S.}$$

**Q8 :**

$$\tan x = -\frac{4}{3}, \ x \text{ in quadrant II}$$

**Answer :**

Here, *x* is in quadrant II.

i.e., $\dfrac{\pi}{2} < x < \pi$

$\Rightarrow \dfrac{\pi}{4} < \dfrac{x}{2} < \dfrac{\pi}{2}$

Therefore, $\sin\dfrac{x}{2}, \ \cos\dfrac{x}{2}$ and $\tan\dfrac{x}{2}$ are all positive.

It is given that $\tan x = -\dfrac{4}{3}$.

$$\sec^2 x = 1 + \tan^2 x = 1 + \left(\dfrac{-4}{3}\right)^2 = 1 + \dfrac{16}{9} = \dfrac{25}{9}$$

$$\therefore \cos^2 x = \dfrac{9}{25}$$

$$\Rightarrow \cos x = \pm\dfrac{3}{5}$$

As $x$ is in quadrant II, $\cos x$ is negative.

$$\therefore \cos x = \dfrac{-3}{5}$$

Now, $\cos x = 2\cos^2\dfrac{x}{2} - 1$

$$\Rightarrow \dfrac{-3}{5} = 2\cos^2\dfrac{x}{2} - 1$$

$$\Rightarrow 2\cos^2\dfrac{x}{2} = 1 - \dfrac{3}{5}$$

$$\Rightarrow 2\cos^2\dfrac{x}{2} = \dfrac{2}{5}$$

$$\Rightarrow \cos^2\dfrac{x}{2} = \dfrac{1}{5}$$

$$\Rightarrow \cos\dfrac{x}{2} = \dfrac{1}{\sqrt{5}} \qquad \left[\because \cos\dfrac{x}{2} \text{ is positive}\right]$$

$$\therefore \cos\dfrac{x}{2} = \dfrac{\sqrt{5}}{5}$$

$$\sin^2\dfrac{x}{2} + \cos^2\dfrac{x}{2} = 1$$

$$\Rightarrow \sin^2\dfrac{x}{2} + \left(\dfrac{1}{\sqrt{5}}\right)^2 = 1$$

$$\Rightarrow \sin^2\dfrac{x}{2} = 1 - \dfrac{1}{5} = \dfrac{4}{5}$$

$$\Rightarrow \sin\dfrac{x}{2} = \dfrac{2}{\sqrt{5}} \qquad \left[\because \sin\dfrac{x}{2} \text{ is positive}\right]$$

$$\therefore \sin\dfrac{x}{2} = \dfrac{2\sqrt{5}}{5}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)} = 2$$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{2\sqrt{5}}{5}$, $\frac{\sqrt{5}}{5}$, and 2.

**Q9 :**

Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ for $\cos x = -\frac{1}{3}$, *x* in quadrant III

**Answer :**

Here, *x* is in quadrant III.

i.e., $\pi < x < \frac{3\pi}{2}$

$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

Therefore, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are negative, whereas $\sin\frac{x}{2}$ is positive.

It is given that $\cos x = -\frac{1}{3}$.

$\cos x = 1 - 2\sin^2\frac{x}{2}$

$\Rightarrow \sin^2\frac{x}{2} = \frac{1 - \cos x}{2}$

$\Rightarrow \sin^2\frac{x}{2} = \frac{1 - \left(-\frac{1}{3}\right)}{2} = \frac{\left(1 + \frac{1}{3}\right)}{2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$

$\Rightarrow \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}}$ $\qquad \left[\because \sin\frac{x}{2} \text{ is positive}\right]$

$\therefore \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

Now, $\cos x = 2\cos^2\frac{x}{2} - 1$

$$\Rightarrow \cos^2 \frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left(-\frac{1}{3}\right)}{2} = \frac{\left(\frac{3-1}{3}\right)}{2} = \frac{\left(\frac{2}{3}\right)}{2} = \frac{1}{3}$$

$$\Rightarrow \cos \frac{x}{2} = -\frac{1}{\sqrt{3}} \qquad \left[\because \cos \frac{x}{2} \text{ is negative}\right]$$

$$\therefore \cos \frac{x}{2} = -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}{\left(\frac{-1}{\sqrt{3}}\right)} = -\sqrt{2}$$

Thus, the respective values of $\sin \frac{x}{2},\ \cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are $\frac{\sqrt{6}}{3},\ \frac{-\sqrt{3}}{3},$ and $-\sqrt{2}$.

**Q10 :**

**Find** $\sin \frac{x}{2},\ \cos \frac{x}{2}$ and $\tan \frac{x}{2}$ **for** $\sin x = \frac{1}{4}$, *x* in quadrant II

**Answer :**

Here, *x* is in quadrant II.

i.e., $\dfrac{\pi}{2} < x < \pi$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Therefore, $\sin \frac{x}{2}, \cos \frac{x}{2}$, and $\tan \frac{x}{2}$ are all positive.

It is given that $\sin x = \dfrac{1}{4}$.

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\Rightarrow \cos x = -\frac{\sqrt{15}}{4} \quad [\cos x \text{ is negative in quadrant II}]$$

$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2} = \frac{1 - \left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4 + \sqrt{15}}{8}$$

$$\Rightarrow \sin \frac{x}{2} = \sqrt{\frac{4 + \sqrt{15}}{8}} \qquad \left[\because \sin \frac{x}{2} \text{ is positive}\right]$$

$$= \sqrt{\frac{4 + \sqrt{15}}{8} \times \frac{2}{2}}$$

$$= \sqrt{\frac{8 + 2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8 + 2\sqrt{15}}}{4}$$

$$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2} = \frac{1 + \left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4 - \sqrt{15}}{8}$$

$$\Rightarrow \cos \frac{x}{2} = \sqrt{\frac{4 - \sqrt{15}}{8}} \qquad \left[\because \cos \frac{x}{2} \text{ is positive}\right]$$

$$= \sqrt{\frac{4 - \sqrt{15}}{8} \times \frac{2}{2}}$$

$$= \sqrt{\frac{8 - 2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8 - 2\sqrt{15}}}{4}$$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\left(\frac{\sqrt{8 + 2\sqrt{15}}}{4}\right)}{\left(\frac{\sqrt{8 - 2\sqrt{15}}}{4}\right)} = \frac{\sqrt{8 + 2\sqrt{15}}}{\sqrt{8 - 2\sqrt{15}}}$$

$$= \sqrt{\frac{8 + 2\sqrt{15}}{8 - 2\sqrt{15}} \times \frac{8 + 2\sqrt{15}}{8 + 2\sqrt{15}}}$$

$$= \sqrt{\frac{\left(8 + 2\sqrt{15}\right)^2}{64 - 60}} = \frac{8 + 2\sqrt{15}}{2} = 4 + \sqrt{15}$$

Thus, the respective values of $\sin \dfrac{x}{2}, \ \cos \dfrac{x}{2} \ \text{and} \ \tan \dfrac{x}{2}$ are $\dfrac{\sqrt{8+2\sqrt{15}}}{4}, \ \dfrac{\sqrt{8-2\sqrt{15}}}{4},$

and $4+\sqrt{15}$